Placement Exam (Foundations Level)

CyberMath Academy

Summer 2022

Instructions (please read!)

Solve as many of the following 23 problems (worth 100 points) as you can in 1 hour. The exam will be split into two sections. Section 1 is the Multiple Choice section, and section 2 is the Short Answer section.

If you obtain the correct answer to a question, you will receive full credit, regardless of whether or not you show your work. On the other hand, even if your answer is incorrect, you still have the chance to earn partial credit if you provide some justification for your answer.

In general, the questions are increasing in difficulty within each section; question 1 is intended to be fairly approachable, whereas the last question is meant to be difficult. Don't let this discourage you from solving questions out of order, though; choose whichever problems you like, and strategize according to whichever you feel that you are most likely to solve within the time limit.

Please also note that this exam has no set "passing" or "failing" score. In particular: *you do not need anywhere near the "traditional" score of 70 percent to pass!!* This is an exam meant to see where you top out and to expose you to new ideas, so don't worry if you haven't seen everything before – after all, that's what the camps are for!

Best of luck!! :)

Section 1: Multiple Choice (60 pts.)

Suggested time: 40 minutes.

Each question in this section ONLY requires the letter (A, B, C, D, E) as the final answer. If you would like, you can show your work to try and receive partial credit in case your answer is incorrect. Each correct answer in this section is worth 4 points. Incorrect answers are not penalized, so it's in your best interest to answer every question.

1. What is the value of $1 + 2 + 3 + 4 + \cdots + 25$?

   (a) 250
   (b) 275
   (c) 300
   (d) 325
   (e) 350

2. How many sevens are in ten ones plus three fifteens plus four sixteens?

   (a) 5
   (b) 6
   (c) 7
   (d) 8
   (e) 9

3. What is the largest positive integer n for which half of n plus one is at least two-thirds of n?

   (a) 4
   (b) 6
   (c) 7
   (d) 8
   (e) 9

4. How many 2×4 squares can fit inside a 12×16 rectangle?

   (a) 24
   (b) 32
   (c) 36
   (d) 48
   (e) 72

5. Today is Tuesday. What day of the week will it be 60 days from today?

   (a) Thursday
   (b) Friday
   (c) Saturday
   (d) Sunday
   (e) Monday

6. I have a bunch of apples. If I sort them into groups of five, I will have two left over, but if I sort them into groups of eight, I will have three left over. What is the fewest number of apples I may have?

(a) 12

(b) 27

(c) 42

(d) 57

(e) 77

7. What is the sum of all numbers that are twice as far away from 24 as from 48?

(a) 40

(b) 72

(c) 96

(d) 112

(e) 136

8. A rectangle has integer side lengths and diagonal length $3\sqrt{10}$. What is its perimeter?

(a) 16

(b) 18

(c) 20

(d) 22

(e) 24

9. How many different ways are there to arrange the letters in the word SUMMER?

(a) 180

(b) 240

(c) 360

(d) 480

(e) 720

10. What is the value of $\frac{27^4}{243^2}$?

(a) 1

(b) 3

(c) 9

(d) 27

(e) 81

11. A *triangular number* is a positive integer that can be written as $1 + 2 + 3 + \cdots + n$ for some positive integer n. How many three-digit triangular numbers are there?

(a) 31

(b) 32

(c) 35

(d) 40

(e) 45

12. A regular polygon has at least 100 interior diagonals. What is the fewest number of sides it may have?

(a) 10

(b) 11

(c) 13

(d) 15

(e) 16

13. The sum of three positive integer multiples of 3 is a positive integer ending in 5. What is the smallest possible product of the three integers?

(a) 72

(b) 81

(c) 90

(d) 108

(e) 120

14. An even positive integer less than or equal to 10, and a positive integer multiple of 3 less than or equal to 10, are selected at random. What is the probability that their sum is a multiple of 6?

(a) $\frac{1}{15}$

(b) $\frac{2}{15}$

(c) $\frac{1}{5}$

(d) $\frac{4}{15}$

(e) $\frac{1}{3}$

15. If we multiply 2 by itself n times, and then divide the result by 105, the remainder will be 1. What is the smallest possible value of n?

(a) 9

(b) 10

(c) 12

(d) 15

(e) 16

Section 2: Short Answer (40 pts.)

Suggested time: 20 minutes.

Each question in this section ONLY requires a number as the final answer. If you would like, you can show your work to try and receive partial credit in case your answer is incorrect. Each correct answer in this section is worth 5 points.

16. What is the total number of digits in all of the question numbers (1-23) on this test?

17. How many positive integers have a square root that is at least 4, but less than 5?

18. How many three-digit positive integers are perfect squares?

19. A rectangle with integer side lengths, in units, has perimeter 22 units. What is its largest possible area, in square units?

20. What is the volume, in cubic units, of a cube whose edge lengths sum to 60 units?

21. What is the exponent of the largest power of 3 that is a factor of 22!?

22. What is the sum of all the possible remainders when a number of the form 2^k, where k is a non-negative integer, is divided by 20?

23. What is the value of

$$1 \cdot 2 + 3 \cdot 4 + 5 \cdot 6 + 7 \cdot 8 + \cdots + 99 \cdot 100?$$